Team Name: | Team #:

Names:

Compound Machines

MIT Science Olympiad Invitational Tournament 2015

1/24/2015 - 50 Minutes

Supervised by Mitchell Gu

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Do not open this packet until we have announced the time has started!

Information

- The event will be **50 minutes**.
- There are **100 independently earnable points** on the written portion:
  - Questions 1-3 are each 10 multiple choice problems worth 1 point each.
  - Questions 4-5 are each 5 short calculation problems worth 2 points each.
  - Questions 6-12 are free-response problems worth 6-9 points each.
- Write all final answers to multiple choice and short answer problems **on the answer sheet.**
- Free Response Questions
  - **Refer to diagram sheet** for all problems.
  - Show all work **in the boxed space.**
  - **Box all requested quantities.**
  - **Show work** for partial credit.
- **Assume ideal conditions unless said otherwise**
- Please use **metric units** in your answers with the appropriate **significant figures**.
- **You may separate the pages of the test**.
- The score out of 100 will be scaled by $\frac{1}{2}$ to obtain the Exam Score (ES) out of 50.
- During the 50 minutes, your team will be called to a station to test your device. (≤ 5 minutes)
- Tiebreakers have the following precedence: exam score, then time score, then selected exam questions.

Point Table
(for grading use only)

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	6	
7	7	
8	7	
9	7	
10	7	
11	8	
12	8	
Total:	100	

1. General concepts.
   (a) (1 point) What characteristic do all simple machines share?
      A. the output work done on the load is greater than the work done by the applied force
      B. decreases the distance that a force must be applied
      C. changes the magnitude or direction of a force
      D. translates a rotational force into a linear one
      E. all of the above
   (b) (1 point) What does it mean for a simple machine to be ideal?
      A. it has no friction or elasticity
      B. its output power is equal to its input power
      C. its mechanical advantage is 1
      D. (a) and (b)
      E. (b) and (c)
   (c) (1 point) Which of the following is equal to the mechanical advantage of a compound machine?
      A. the product of the mechanical advantages of the simple machines that compose it
      B. the sum of the mechanical advantages of the simple machines that compose it
      C. the maximum of the mechanical advantages of the simple machines that compose it
      D. the minimum of the mechanical advantages of the simple machines that compose it
      E. must be determined empirically.
   (d) (1 point) What is actual mechanical advantage?
      A. the ratio of the measured output force to the measured input force
      B. the ratio of the measured output power to the measured input power
      C. the ratio of the measured output velocity to the measured input velocity
      D. the ratio of the calculated output force to the calculated input force, assuming no energy loss
      E. the ratio of the calculated output power to the calculated input power, assuming no energy loss
   (e) (1 point) Which of the following are true of machines in static equilibrium?
      A. the net force and net torque on the machine is zero
      B. the linear and rotational momentum are zero
      C. the mechanical advantage is 1
      D. (a) and (b) only
      E. all of the above

(f) (1 point) How is the efficiency of a simple machine defined?

A. the ratio of the ideal mechanical advantage to the actual mechanical advantage
B. the ratio of the work output to the work input
C. the ratio of the force output to the force input
D. the ratio of the power output to the power input
E. none of the above.

(g) (1 point) What does it mean for a simple machine to be *self-locking*?

A. The machine will stop itself once a certain amount of work has been done.
B. The machine will stop itself once the load has been moved a certain distance.
C. Under a certain threshold torque, the machine won't move.
D. The machine has a limited range of motion.
E. No amount of load force can cause the machine to move backwards against the applied force

(h) (1 point) What quantity is always preserved in simple and compound machines?

A. Force
B. Torque
C. Velocity
D. Work
E. Friction

(i) (1 point) Which simple machines involve the application of torque?

I Lever
II Wheel and Axle
III Pulley
IV Screw
V Wedge

A. I only
B. I and V only
C. II and IV only
D. I, II, III, and IV only
E. I, II, III, IV, and V

(j) (1 point) What are the SI units of force, work, and torque, respectively?

A. Newton, Joule, Newton-meter
B. Newton, Joule, Newton/meter,
C. Newton, Joule/second, Newton-meter
D. Joule, Newton-second, Newton/meter
E. Joule, Watt, Newton/meter

2. Levers, inclined planes, and wedges.

(a) (1 point) What is the ideal mechanical advantage formula for a lever?

A. $\frac{\text{effort arm length}}{\text{load arm length}}$

B. $\frac{\text{load arm length}}{\text{effort arm length}}$

C. (load arm length) $\cdot$ (effort arm length)

D. (load arm length) $-$ (effort arm length)

E. Depends on the lever class

(b) (1 point) What is the ideal mechanical advantage formula for an inclined plane?

A. $\frac{\text{height}}{\text{horizontal length}}$

B. $\frac{\text{height}}{\text{hypotenuse}}$

C. $\frac{\text{horizontal length}}{\text{height}}$

D. $\frac{\text{horizontal length}}{\text{hypotenuse}}$

E. $\frac{\text{hypotenuse}}{\text{height}}$

(c) (1 point) What is the mechanical advantage formula for a wedge?

A. $\frac{\text{thickness}}{\text{side length}}$

B. $\frac{\text{side length}}{\text{thickness}}$

C. $\tan\theta$, where θ is the wedge angle

D. $\tan\frac{\theta}{2}$

E. $\cos\theta$

(d) (1 point) Of levers, inclined planes, and wedges, which are likely to be self-locking?

A. Levers only

B. Inclined planes only

C. Levers and wedges only

D. Inclined planes and wedges only

E. Levers, inclined planes, and wedges

(e) (1 point) For a lever to be in static equilibrium,

A. The net torque must be zero.

B. The lengths of the effort arm and the load arm must be equal.

C. The applied force must equal the output force.

D. The output force must be zero.

E. None of the above.

(f) (1 point) The following would increase a wedge's mechanical advantage:

A. Increase thickness and increase length

B. Increase thickness and decrease length

C. Decrease thickness and increase length

D. Decrease thickness and decrease length

(g) (1 point) The following would increase a lever's mechanical advantage:

A. Increase the load arm length

B. Increase the effort arm length

C. Double both the load and effort arm length

D. Halve both the load and effort arm length

E. None of the above

(h) (1 point) In rowing, rowers propel a boat by pulling a paddle through the water. What class of lever does the paddle resemble?

A. Class I

B. Class II

C. Class III

(i) (1 point) Suppose a rower's arms are getting tired and wishes to decrease the force he must exert without changing the force applied to the water. What could he do?

A. Shift his grip away from the fulcrum.

B. Shift his grip towards the fulcrum.

C. Make shorter strokes through the water.

D. Make longer strokes through the water.

(j) (1 point) After following your advice from the previous problem, the rower wants to maintain the same work per stroke that he expended previously. What compromise must he make?

A. Shift his grip away from the fulcrum.

B. Shift his grip towards the fulcrum.

C. Make shorter strokes through the water.

D. Make longer strokes through the water.

3. Wheels, screws, pulleys.

(a) (1 point) What is the mechanical advantage formula for a wheel/axle?

A. $\frac{\text{wheel radius}}{\text{axle radius}}$

B. $\frac{\text{axle radius}}{\text{wheel radius}}$

C. $\frac{(\text{wheel radius})^2}{(\text{axle radius})^2}$

D. $\frac{(\text{axle radius})^2}{(\text{wheel radius})^2}$

(b) (1 point) What is the mechanical advantage formula for a gear?

A. $\frac{\text{radius of input gear}}{\text{radius of output gear}}$

B. $\frac{\text{number of teeth on output gear}}{\text{number of teeth on input gear}}$

C. $\frac{\text{rotational velocity of output gear}}{\text{rotational velocity of input gear}}$

D. $\frac{(\text{radius of output gear})^2}{(\text{radius of input gear})^2}$

(c) (1 point) What is the mechanical advantage formula for a screw?

A. $\frac{\text{diameter of screw}}{\text{pitch of screw thread}}$

B. $\frac{\text{circumference of screw}}{\text{pitch of screw thread}}$

C. $\frac{\text{pitch of screw thread}}{\text{diameter of screw}}$

D. $\frac{\text{pitch of screw thread}}{\text{circumference of screw}}$

(d) (1 point) What is the mechanical advantage formula for a single movable pulley?

A. 1

B. 2

C. $\frac{\text{length of rope to load}}{\text{length of rope to effort}}$

D. $\frac{\text{length of rope to effort}}{\text{length of rope to load}}$

(e) (1 point) What is the mechanical advantage formula for a belt?

A. 1

B. 2

C. $\frac{\text{radius of input pulley}}{\text{radius of output pulley}}$

D. $\frac{\text{radius of output pulley}}{\text{radius of input pulley}}$

(f) (1 point) What is a typical efficiency for a screw?

A. 0

B. 0.12

C. 0.33

D. 0.67

E. 0.95

(g) (1 point) The mechanical advantage of a system of pulleys can quickly be determined by:

A. Doubling the number of anchored pulleys.
B. Doubling the number of movable pulleys.
C. Doubling the total number of pulleys.
D. Calculating the ratios of the lengths of rope between each pulley

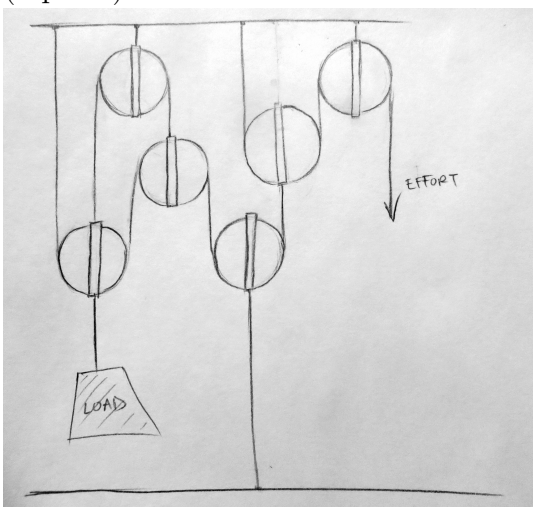
(h) (1 point) What is the mechanical advantage of a worm gear/worm pair?

A. $\frac{\text{worm gear radius}}{\text{worm radius}}$
B. $\frac{\text{pitch of the worm gear}}{\text{pitch of the worm}}$
C. $\frac{\text{number of teeth on the worm gear}}{\text{number of teeth on the worm}}$
D. Number of teeth on the worm
E. Number of teeth on the worm gear

(i) (1 point) When a cyclist shifts to a lower gear to pedal up a steep incline, they are primarily:

A. Increasing their mechanical advantage
B. Decreasing their mechanical advantage
C. Increasing their efficiency
D. Decreasing their efficiency

(j) (1 point) Find the ideal mechanical advantage of the following pulley system.



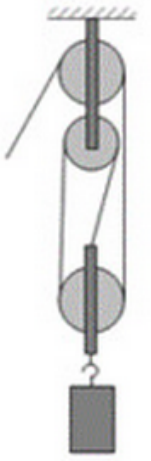
A. 2
B. 3
C. 4
D. 5
E. 6

4. (10 points) Short calculation, 2pts per question.

   (a) (2 points) A gear train is composed of three gears A, B, and C, which have 48, 18, and 36 teeth, respectively. Gears A and B share a shaft, and gear C is meshed with gear B. Assume gear A is the input and gear C the output of the system. What is the mechanical advantage of the system?

   (b) (2 points) In order to push a 480N couch up a frictionless ramp and into a U-haul truck at a constant rate, Mitchell must apply 200.N of force. If the base of the U-haul truck is 1.0 meter off the ground, how long is the ramp?

   (c) (2 points) Jingyi is moving 56N of pad thai in a wheelbarrow. The bucket holding the food is 0.30 meters from the wheel, and the handles of the wheelbarrow are 0.50 meters from the bucket. When she is standing still, how much force does she need to exert on the handles in order to lift the pad thai?



   (d) (2 points)
   In the pulley system above, a force is applied to the end of the rope so that the 12.0kg mass is being moved up at a constant rate. If the tension in the rope is 50.0N, what is the efficiency of the pulley system?

   (e) (2 points) On top of a well, a bucket is attached to a rope wound around a cylinder of diameter 18cm, and a crank arm of radius 33cm is turned in order to lower and raise the bucket. What is the ideal mechanical advantage of this machine?

5. (10 points) Short Calculation, 2pts per question.

   (a) (2 points) The MacBook Air is notorious for its wedge design. According to Apple's website, the Air's thickness at most is 1.7cm and its wedge length is 22.7 cm. What is the Air's mechanical advantage?

   (b) (2 points) For ice skaters, wedges on the blade of each skate are important for the skater's stability. If a skater's skates are slipping on the ice, what adjustment needs to be made to the blade and why? (in terms of mechanical advantage)

   (c) (2 points) A certain bolt has diameter 4.0mm and length 1.6cm and takes 24 full revolutions to tighten into place. What is the greatest actual mechanical advantage the bolt can have for it to be self-locking?

   (d) (2 points) A log of uniform diameter and density is resting on a rock so that, when Joe sits at one end of the log, the log perfectly balances on top of the rock. If Joe weighs 60.kg, and the rock is three times farther from the other end of the log than it is from him, how much does the whole log weigh, in kg?

   (e) (2 points) A snowball of mass 8kg rolls up a straight slope of incline angle 30.°. At the base of the slope the snowball has velocity 6.00 m/s, and it rolls 4.00m along the slope before rolling back down. What is the coefficient of friction of the slope? Assume $g = 10m/s^2$.

6. (6 points) Belt Drive Turntable

   Many modern vinyl turntables are belt driven; a motor with a small pulley is coupled to a much larger record platter with a belt. For the audio to have the right tempo and pitch, it's critical that the platter spins at $33\frac{1}{3}$ rpm (for most records).

   Suppose a turntable has a platter pulley radius of 12.0 cm and its drive pulley has a radius of 1.00cm. Calculate:

   1. The speed in rpm that the motor must rotate at.
   2. The mechanical advantage of the belt system.
   3. The required torque of the motor given that the belt must apply 2.00 N of force to the platter for it to spin at a constant $33\frac{1}{3}$ rpm.

7. (7 points) Guitar Tuner

To hold guitar strings at the right tension, guitar tuning machines use a compound machine with a spur gear, a worm gear, and a wheel/axle (the knob). A particular tuning machine has a string peg radius of 3.00mm (attached to the spur gear), a spur gear with 14 teeth, and a knob radius of 9.00mm.

1. Calculate the gear ratio of the worm gear system.
2. When the string is tuned with 3.84 Newtons of force on the ends of the knob, the string's tension is 100. Newtons. Calculate the efficiency of the tuning machine system.
3. Suppose a string is in tune at a tension of 80.0 Newtons and you apply a lubricant that increases the efficiency found above by 0.200. Calculate the force in newtons you must apply to the ends of the knob.

8. (7 points) Bungee Drop

   Your Bungee Drop teammates need help! They ask you to devise a clamp to anchor their bungee device that can be hand-tightened to the correct pressure for maximum convenience. They've determined that the clamp must exert 210. N of force on the bungee device to hold it and request that it can be tightened with only 5.00 N of effort. The clamp will use a lever coupled to a screw, similar to many vises. The only screw available for the clamp has an outside diameter of 12.0mm and a pitch of 2.0mm. Calculate the minimum lever length needed to satisfy their requirements.

9. (7 points) Nail Clipper

   Nail clippers can be represented as a compound machine of two levers and a wedge that cleaves the fingernail.

   1. Using the specifications found in the diagram, calculate the mechanical advantage of each simple machine and also the entire compound machine.
   2. Assuming it takes 25.0 Newtons of lateral force on a fingernail to cleave it, how much effort must a user apply to cut their fingernails?

10. (7 points) Elevator Counterweights

To lower the load on the motor in a traditional elevator, a counterweight is used on one end of a pulley system to balance with the elevator cab.

For the elevator pulley system pictured, the elevator cab has a mass of 500. kg and must have a passenger capacity of 2.00×10^3 kg.

1. Find a counterweight mass that minimizes the motor load at half the passenger capacity.
2. For a lift motor with a 10.0 cm pulley radius, what is the maximum torque required to allow static equilibrium at all capacities?

11. (8 points) Crane counterweights

In construction cranes, the main horizontal jib must be counter-weighted on the opposite side of the load so that the crane doesn't collapse at the vertical support point. You are designing a crane that must be able to lift a 5.00×10^3 kg load 70.0 meters away from the vertical support. However, the vertical support can withstand 2.00×10^6 newton meters of torque before failing. Use 9.81 as the acceleration of gravity, g.

1. What is an appropriate counterweight mass needed if the counterweight is mounted 20.0m behind the vertical support? The crane must not collapse under no load as well.
2. Using the counterweight mass found above, what is the maximum load mass the crane can support 20.0m in front of the vertical support?

12. (8 points) Mission Possible

Your Mission Possible teammates need help as well. Their device needs to easily lift golf balls a certain distance, but they're unsure what compound machine would be best.They need to convert rotational motion from a motor to vertical linear motion and have mechanical advantage, so you settle on a screw conveyor and inclined plane design, which can accommodate lifting many balls at once.

For the screw conveyor, the screw has an overall radius of 10.0cm and a pitch (thread spacing) of 5.00 cm. The motor is directly coupled with this screw and can only output 1.79 mN m of torque. Your goal is to be able to lift five 45.9 gram golf balls at once at static equilibrium.

1. What is the maximum angle for the inclined plane where this is possible?
2. Find the mechanical advantage of the screw, the inclined plane, and the whole system.