TROY INVITE KEY

Sound of Music 2018 / 2019

Show all work! $I_0 = 10^{-12}\ \frac{W}{m^2}$

1. A particular piece of heavy machinery emits a sound level of 77.0 dB when measured at a distance of 10.0 m from the device. If 132 machines operate at the same time in close proximity to one another, at what distance from the machinery will the combined sound level be 85.0 dB?

$$\beta = 10 \log_{10} \frac{I}{I_0}$$

$$I = I_0 10^{(\beta/10)}$$

IF $\beta = 77.0\,dB$

$$I = 5.01 \times 10^{-5}\ \frac{W}{m^2}$$

IF $\beta = 85.0\,dB$

$$I = 3.16 \times 10^{-4}\ \frac{W}{m^2}$$

FOR ISOTROPIC SOURCE

$$I = \frac{P}{4\pi r^2}$$

ONE MACHINE

$$P = 4\pi r^2 I$$

$$= 4\pi (10.0\,m)^2 (5.01 \times 10^{-5}\ \tfrac{W}{m^2})$$

$$P_{ONE} = 0.0630\,W$$

$$P_{TOTAL} = (132)(P_{ONE})$$

$$P_{TOTAL} = 8.31\,W$$

$$r^2 = \frac{P}{4\pi I}$$

$$r = \sqrt{\frac{8.31\,W}{4\pi (3.16 \times 10^{-4}\ \frac{W}{m^2})}}$$

$$\boxed{r = 45.7\,m}$$

2. A clarinet plays a note of 523.3 Hz, and is considered "in tune". Another player plays the same note, but is told his instrument is 14 cents sharp. What is the beat frequency heard if the two players play simultaneously?

$$\frac{f_2}{f_1} = 2^{\left(\frac{¢}{1200}\right)}$$

14 ¢ SHARP $\Rightarrow$ ¢ $= +14$

$$\frac{f_2}{f_1} = 2^{\frac{14}{1200}}$$

$$f_2 = f_1 \cdot 2^{\frac{14}{1200}}$$

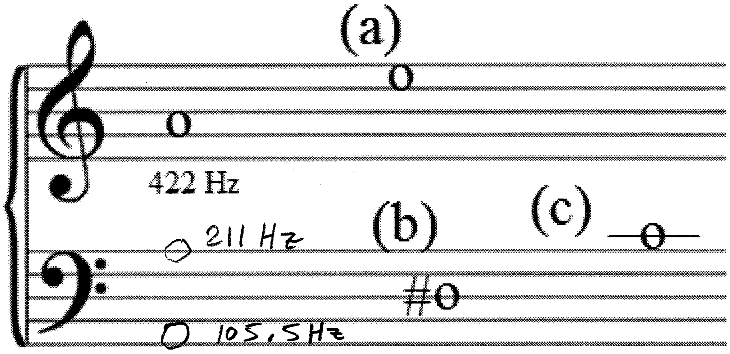
$$f_{BEAT} = |f_2 - f_1|$$

$$= f_1 \left[2^{\frac{14}{1200}} - 1 \right]$$

$$= (523.3\,Hz)\left[1.00812 - 1\right]$$

$$\boxed{f_{BEAT} = 4.25\,Hz}$$

3. An orchestra tunes to the note shown on the left side of the staff. This note is set to be 422 Hz (as shown).
(a) Identify the note labeled (a) in the diagram and determine its equal tempered frequency.



e (7 ½-STEPS UP FROM a)

$$f = (422\,Hz)(2^{7/12}) = 632\ Hz$$

(b) Identify the note labeled (b) in the diagram and determine its equal tempered frequency.

6 ½-STEPS UP FROM a = 105.5 Hz

d#

d-sharp $\quad f = (105.5\,Hz)(2^{6/12}) = 149\ Hz$

(c) Identify the note labeled (c) in the diagram and determine its equal tempered frequency.

3 ½-STEPS UP FROM a = 211 Hz

C

$$f = (211\,Hz)(2^{3/12}) = 251\ Hz$$

(d) Identify the tuning note.

a

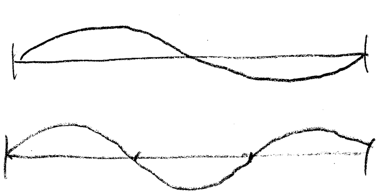
4. A violin string 30.0 cm long with linear density 0.650 g/m is placed near a loudspeaker that is fed by an audio oscillator of variable frequency. The frequency of the oscillator is swept through a frequency range from 500 Hz to 1500 Hz. It is found that the string is set into oscillation only at frequencies of 880 Hz and 1320 Hz. What is the tension in the string?

$$\frac{1320\,Hz}{880\,Hz} = 1.5$$

3:2 RATIO

880 Hz IS 2nd HARMONIC

1320 Hz IS 3rd HARMONIC



$\lambda = L = 30.0\,cm$

$\lambda = \frac{2}{3}L = 20.0\,cm$

$v = \lambda f = (0.300\,m)(880\,Hz)$

$v = 264\ m/s$

$$v = \sqrt{\frac{F}{\mu}}$$

$$F = \mu v^2$$

$$F = \left(\frac{0.650\times10^{-3}\,kg}{m}\right)\left(264\,\frac{m}{s}\right)^2$$

$$\boxed{F = 45.3\ N}$$

5. Two identical closed pipes sound at their fundamental frequency and produce a frequency of 196.~~0 Hz~~. At 0°C, the speed of sound is 331 m/s. (a) What is the length of the closed pipe that produces this frequency sound?

CLOSED PIPE $\lambda_{n=1} = 4L$

$$\lambda f = v$$

$$\lambda = \frac{331\ m/s}{196.0\ Hz} = 1.69\ m \qquad L = \frac{\lambda}{4} = 0.422\ m$$

(b) One pipe is exposed to direct sunlight, which warms the pipe. When the two pipes sound together now, a beat frequency of 3.4 Hz is heard. What is the frequency of the warmer pipe?

$$f_B = |f_1 - f_2| \qquad f_{WARM} = f_{COLD} + 3.4\ Hz$$

WARMING PIPE INCREASES SPEED OF SOUND, INCREASING THE FREQUENCY.

$$f_{WARM} = 199.4\ Hz$$

(c) If the speed of sound is directly proportional to the square root of the absolute temperature of the air column, what is the temperature inside the warm pipe, in °C?

$$v \propto \sqrt{T}, \quad v = \lambda f$$

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

THERMAL EXPANSION OF PIPE IS NEGLIGIBLE

$v = \lambda f$, λ = CONST.

$$\frac{T_2}{273\ k} = \left(\frac{199.4\ Hz \cdot \lambda}{196.0\ Hz \cdot \lambda}\right)^2$$

$$T_2 = 283\ k = 10°C$$

(d) How many cents sharp is this pipe due to the warming?

$$¢ = 1200 \frac{\ln\left(\frac{f_2}{f_1}\right)}{\ln 2} = 29.8 \text{ CENTS SHARP}$$

6. Becky stands next to a parked car. The driver honks his horn at her, and Becky hears a frequency of 600 Hz from the horn. As the driver leaves, he honks the horn again, and Becky hears a frequency of 585 Hz. What is the speed of the car? The speed of sound at this location is 345 m/s.

$$f_s = 600\ Hz$$

$$f_o = f_s \frac{v \pm \cancel{v_o}^{\to 0}}{v \pm v_s}$$

OBSERVER AT REST

SOURCE MOVES AWAY, $f_o < f_s$

$$f_o = f_s \frac{v}{v + v_s} \qquad v_s = ?$$

$$v + v_s = \frac{f_s}{f_o} v$$

$$v_s = \left(\frac{f_s}{f_o} - 1\right) v$$

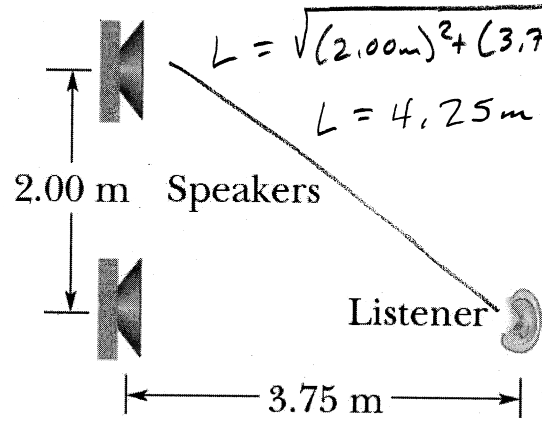
$$v_s = \left(\frac{600\ Hz}{585\ Hz} - 1\right)\left(345\ \tfrac{m}{s}\right)$$

$$\boxed{v_s = 8.85\ Hz}$$

$v = \lambda f$ $\quad f_{min} = 20\ Hz \rightarrow \lambda_{max} = 17.25\,m$

$v_{sound} = 345\ m/s$ $\quad f_{max} = 20\ kHz \rightarrow \lambda_{min} = 0.01725\,m$

7. In the figure below, two loudspeakers, separated by a distance of 2.00 m, are in phase. Assume the amplitudes of the sound from the speakers are approximately the same as the position of the listener, who is 3.75 m directly in front of one of the speakers.
(a) For what lowest frequency in the audible range does the listener hear a minimum signal?



$L = \sqrt{(2.00\,m)^2 + (3.75\,m)^2}$

$L = 4.25\,m$

FOR LOWEST FREQUENCY, OR LONGEST WAVELENGTH, THE DIFFERENCE IN PATH LENGTHS MUST BE $\frac{1}{2}\lambda$.

$\frac{1}{2}\lambda = 0.50\,m$

$\lambda = 1.00\,m$

$\boxed{f = 345\ Hz}$

(b) For what highest frequency in the audible range does the listener hear a maximum signal?

MAXIMUM SIGNAL MEANS PATH LENGTHS ARE $n\lambda$ DIFFERENT, n = INTEGER.

FIND MAXIMUM n: $\quad n(0.01725\,m) = 0.50\,m$

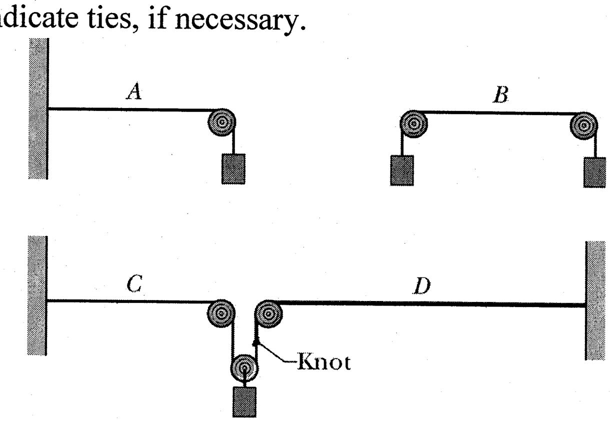
$n = 28.98$

ACCEPT EITHER $n = 28$ OR $n = 29$ FOR ROUNDING DIFFERENCE

$\lambda = \frac{0.50\,m}{29} = 0.01724\,m$ $\quad$ OR $\quad \lambda = \frac{0.50\,m}{28} = 0.01786\,m$

$f = 20.0\ kHz$ $\quad\quad f = 19.32\ kHz$

8. In the diagram below, four strings are placed under tension by one or two suspended blocks, all of the same mass. Strings A, B, and C have the same linear density, while string D has a greater linear density. Rank the strings according to the speed that waves will have when sent along them, greatest first. Indicate ties, if necessary.



$v = \sqrt{\frac{F_T}{\mu}}$

A & B TIE, SAME $F_T = mg$

$v_A = v_B = \sqrt{\frac{mg}{\mu}}$

C & D EACH SUPPORT HALF OF THE WEIGHT mg

$v_C = \sqrt{\frac{\frac{1}{2}mg}{\mu}}$, SO $v_C < v_A$

SINCE $\mu_D > \mu_C$, $v_D < v_C$

$\underbrace{v_A \,\&\, v_B}_{\text{TIE, GREATEST}}$, v_C, $\underbrace{v_D}_{\text{LEAST}}$

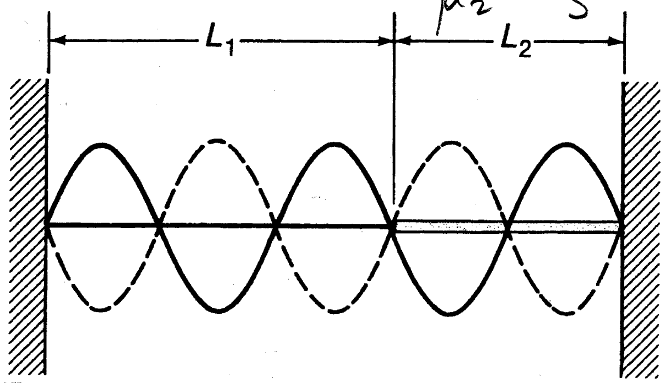
$$\lambda f = v \qquad v = \sqrt{F/\mu}$$

9. Two strings with different linear mass densities are joined together and stretched between two fixed supports. The string system is set into vibration with a standing wave pattern shown in the diagram. Determine the length ratio L_2/L_1 if $\mu_2/\mu_1 = 5/4$.

$$\frac{\mu_1}{\mu_2} = \frac{4}{5}$$

SAME TENSION FORCE

SAME FREQUENCY



$$\lambda_1 = \frac{2}{3} L_1 \rightarrow L_1 = \frac{3}{2}\lambda_1$$

$$\lambda_2 = L_2$$

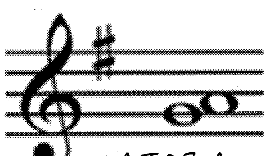
$$\frac{L_2}{L_1} = \frac{\lambda_2}{\frac{3}{2}\lambda_1} \cdot \frac{f}{f} = \frac{v_2}{\frac{3}{2} v_1} = \frac{2 v_2}{3 v_1}$$

$$\frac{L_2}{L_1} = \frac{3 v_2}{2 v_1} = \frac{3\sqrt{F/\mu_2}}{2\sqrt{F/\mu_1}} = \frac{3}{2}\sqrt{\frac{\mu_1}{\mu_2}} = \frac{3}{2}\sqrt{\frac{4}{5}} = \frac{3}{\sqrt{5}}$$

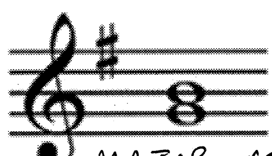
$$\boxed{\frac{L_2}{L_1} = \frac{3}{\sqrt{5}}}$$

10. Label each of the following intervals (number and kind). What major key signature is this?

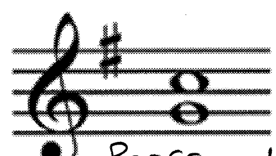
G MAJOR



MAJOR 2nd



MAJOR 3rd



PERFECT 4th

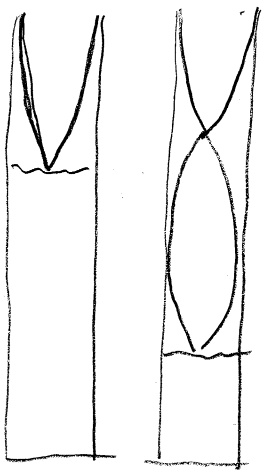
PERFECT 5th

MAJOR 6th

MAJOR 7th

OCTAVE

11. The water level in a vertical glass tube 1.00 m long can be adjusted to any position in the tube. A tuning fork vibrating at 686 Hz is held just over the open top of the tube. At what positions of the water level will there be resonance? Take the speed of sound to be 345 m/s.



RESONANCE OCCURS WHEN AIR CAVITY IS $\frac{1}{4}\lambda$, $\frac{3}{4}\lambda$, $\frac{5}{4}\lambda$, etc. IN LENGTH.

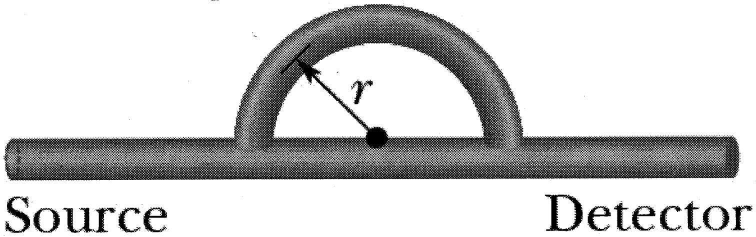
$$f\lambda = v, \quad \lambda = \frac{345\ \text{m/s}}{686\ \text{Hz}} = 0.503\ \text{m}$$

LET
$$\frac{\left(\begin{smallmatrix}\text{LARGEST}\\ \text{VALUE OF } n\end{smallmatrix}\right)\lambda}{4} = 1.00\ \text{m}$$

$$\begin{smallmatrix}\text{LARGEST}\\ \text{OF } n\end{smallmatrix}\ \text{VALUE} = 7.95 = 7 \quad \text{INTEGERS ONLY}$$

$L = 0.126\ \text{m},\ 0.377\ \text{m},\ 0.629\ \text{m},\ 0.880\ \text{m}$, FOUR SOLUTIONS!

12. A sound wave of 863 Hz enters the tube shown in the figure below at the source. What must be the smallest radius r such that the minimum will be heard at the detector end? Take the speed of sound to be 345 m/s.



SMALLEST r CORRESPONDS TO $\frac{1}{2}\lambda$ DIFFERENCE IN PATH LENGTH.

$$\pi r - 2r = \frac{1}{2}\lambda = \frac{1}{2}\frac{v}{f}$$

$$r = \frac{v}{2(\pi - 2)f} = 0.175\ \text{m}$$