

Technical Problem Solving 2014-2015

This exam is not guaranteed to be error free, but attempts to cover the main topics of the event. In particular, the following topics are covered:

- Newton's law of cooling
- Model fitting
- Residuals
- Correlation coefficient
- Outliers
- Mean, median mode
- Standard deviation
- Normal distribution
- Random/systematic error

Please post comments/questions/corrections on scioly.org forums under 2015 "Technical Problem Solving C", direct link here:

<http://www.scioly.org/phpBB3/viewtopic.php?f=186&t=5917>

If there are any corrections, they will be posted there.

This was developed for Santa Clara County Regional Science Olympiad.

The solutions may not have the correct significant figures.

Technical Problem Solving

Division C
March 21, 2015
Bay Area Regional Science Olympiad

Team name: _____
Team number: _____
Team member name #1: _____
Team member name #2: _____

Time: 50 minutes

Scoring:

- 200 points total, 100 points for each of two stations
- each question is worth 10 points
- partial/full credit will be assigned based on both correctness of answers and procedure leading to answers, so please show all work and provide appropriate units and significant figures for answers
- specifying measurement or calculation uncertainty (for example, 1.1 cm +/- **0.1 cm**) is not required unless specifically asked for in the question
- ties will be broken at random

Materials you may use:

- up to two calculators
- Two 8.5" x 11" sheets of paper with information on both sides

Equations:

Newton's law of cooling:

$$T(t) = T_s + (T_o - T_s) \cdot \exp(-kt)$$

Notes

- Please turn phones off
- This may be a difficult test, so do your best
- Good luck!

Station #1

Storyline: Someone was found immobilized shortly after they had started baking a piece of salmon in their oven. When investigators arrived, they noted the oven was at 450F and that the temperature of the piece of salmon was 375F.

Your lab oven only goes up to 350F so you set it at that and let it warm up. You stick your temperature probe in the similarly sized piece of salmon, initially at room temperature (70F), and start the temperature recording at time $t = 6$ minutes. You then place the piece of salmon in the oven, and come back about 5 minutes later and pull out the temperature probe. At this point you stop the temperature recording.

Here is the data:

time (m)	temperature (F)
6	63.7
7	116.6
8	162.5
9	203.2
10	222.7
11	254.4
12	65.5

1. Plot the temperature as a function of time

2. We know from Newton's Law of Cooling that temperature does not change linearly with time. In order to use linear regression, we need to transform the data. Plot the $\ln(\text{temperature}/(350\text{F}-70\text{F}))$ as a function of time (minutes), where $\ln(x)$ denotes the natural logarithm function.

You hand this data over to your two colleagues who come up with two models that describe the data, where Y is the quantity plotted in #2 and t is time in minutes and units have been omitted.

model A: $Y = 0.0699t - 1.3433$

model B: $Y = 0.1875t - 2.1005$

They disagree about who has the better model, so you decide to do some analysis to find out.

3. Compute the residual for each time point using the log of the temperature as defined in #2 and model A. Then plot these residuals.

4. Compute the residual for each time point using the log of the temperature as defined in #2 and model B. Then plot these residuals.

5. Compute the R^2 -values for each model including all time points. When is it possible for the R^2 to be negative?

Model A R^2 : _____

Model B R^2 : _____

6. Which model has the higher R^2 -value, and what does this mean?

7. Which model is more informative for the investigation?

8. Pick the more informative model based on #7. Assume this model gives you the coefficient “k” in Newton’s law of cooling – that is, assume $k = 0.0699$ for model A and $k = 0.1875$ for model B. Determine how many minutes prior to the arrival of the investigators was the piece of salmon placed into the oven. You may assume that the oven was preheated to 450F and that the initial temperature of the salmon was the same as in the lab test.

Answer : _____

9. Name 3 factors that physically impact heating rates in these situations that have not been factored in to our lab experiment.

A.

B.

C.

10. You learn that the temperature probe used in your lab experiments was always 5F higher than the true temperature. However the probe used at the crime scene was still accurate. What type of measurement error is this? How would it qualitatively change your estimate in #8?

Station #2

You take these temperature measurements (F) at time zero:

32, 56, 102, 101, 102, 98, 105, 103

1. Compute the mean, mode and median of the data

mean : _____

mode : _____

median : _____

2. Compute the (population) standard deviation of the data

Standard deviation: _____

3. Unrelated to #1 and #2, draw an ideal normal distribution, and label the mean, standard deviation and label the axes

At a later time exactly one hour later, you take these measurements (units of F):

26, 87, 238, 241, 252, 237, 240, 241

4. Compute the mean, mode and median of the data

mean : _____
mode : _____
median : _____

5. Using the mean temperatures, determine how long after the initial measurement will the temperature reaches 400F, assuming the ambient temperature is 500F.

Answer: _____

6. Do the same, but use the median instead of the mean.

Answer: _____

7. You learn that your temperature probe takes time to start up and reads a much lower temperature when it is initially first powered on, making the first two measurements in each sequence inaccurate. Unfortunately, this happened both times you took the measurements. Without necessarily doing any math, explain which of your answers, to question #5 or #6, is probably more accurate given this information?

8. What is the systematic error in this experiment?

9. What is the random error in this experiment and describe how it could be reduced through statistical analysis?

10. You find a wormhole behind your bookshelf and end up in a world where instead of the rate of change of temperature of a body being proportional to the difference between its own and the surrounding temperature (Newton's law of cooling), it is proportional to the square of that difference. Derive Newton's law of cooling on this new planet.