SECTION C
SOLUTION MANUAL

This solution manual shows my calculations for how I came to my answers in part C. I was too lazy to transcribe it, so it is handwritten. If you have any questions about how I came to my answers, and/or can't read my handwriting, feel free to contact me (some info is given on the exam).

1 SQUARE = ____________

Part C Solutions

29. (a) i. Silicon

ii. As time goes on, we could look at the light curve, and see if it matches the light curve of a type Ia Supernova.

iii. $m = 16.2, M = -19.3$

$$m - M = -5 + 5\log(d)$$
$$16.2 - -19.3 = -5 + 5\log(d)$$
$$35.5 = -5 + 5\log(d)$$
$$8.1 = \log(d)$$
$$10^{\wedge}8.1 = d$$

$\boxed{1.26 \times 10^8\ pc}$

(b) i. Small-angle formula using distance from (a).iii.

$$\theta = 206265\left(\frac{D}{d}\right)$$

Angular Radius

$$\theta = 206265\left(\frac{26100\ pc}{1.26 \times 10^8\ pc}\right) = 42.73'' * 2 = \boxed{85.46''}$$

ii. We can use a doppler s. . side of the galaxy to give us this

$$\text{Absolute Value of spectral shift} = \frac{v}{c} = \frac{254\ km/s}{3.00 \times 10^5\ km/s} = 8.4667 \times 10^{-4}$$

Receding Side:

$$21.1061\ cm(8.4667 \times 10^{-4}) + 21.1061\ cm = 21.124\ cm$$

Approaching Side:

$$21.1061\ cm - 21.1061\ cm(8.4667 \times 10^{-4}) = 21.088\ cm$$

$\boxed{21.088\ cm \text{ to } 21.124\ cm}$

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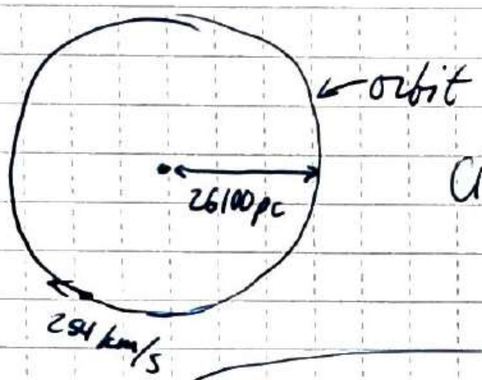
iii.



Circumference of orbit $= 2(26100\,pc)\pi$
$= 163991\,pc$

$$163991\,pc\left(\frac{3.09\times10^{13}\,km}{pc}\right)\left(\frac{s}{254\,km}\right) = \boxed{1.995\times10^{16}\,s}$$

(c) i. This one's a little weirder. First, we want to find the total mass (using our Kepler's third law). Then we want to subtract out the given luminous matter.

$$1.995\times10^{16}\,s\left(\frac{yr}{3.157\times10^{7}\,s}\right) = 6.319\times10^{8}\,yr$$

$$26100\,pc\left(\frac{3.09\times10^{13}\,km}{pc}\right)\left(\frac{AU}{1.496\times10^{8}\,km}\right) = 5.391\times10^{9}\,AU$$

$$p^2 = \frac{a^3}{m} \Rightarrow m = \frac{a^3}{p^2} \Rightarrow m = \frac{(5.391\times10^{9}\,AU)^3}{(6.319\times10^{8}\,yr)^2} = \frac{1.567\times10^{29}\,AU^3}{3.991\times10^{17}\,yr^2}$$

$= 3.926\times10^{11}\,M_\odot$ ← Total Mass

Given luminous mass ↓

$$3.926\times10^{11}\,M_\odot - 1.58\times10^{11}\,M_\odot = \boxed{2.346\times10^{11}\,M_\odot}$$

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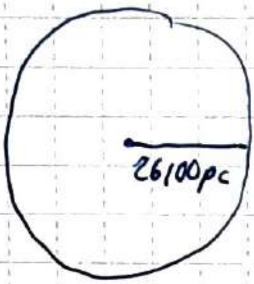
ii.



$$26100\,pc\left(\frac{3.09\times10^{13}\,km}{pc}\right) = 8.065\times10^{17}\,km = r$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(8.065\times10^{20}\,m) = \underline{2.197\times10^{63}\,m^3 = V}$$

$$2.346\times10^{11}\,M_\odot\left(\frac{1.99\times10^{30}\,kg}{M_\odot}\right) = \underline{4.669\times10^{41}\,kg}$$

$$\frac{4.669\times10^{41}\,kg}{2.197\times10^{63}\,m^3} = \boxed{2.125\times10^{-22}\,kg/m^3}$$

iii. First, we need to calculate the luminosity of the galaxy (using the apparent magnitude and distance).

$$15.34 - M = -5 + 5\log(1.26\times10^{8}\,pc)$$
$$M = -20.10$$

$$4.83 + 20.10 = 2.5\log\left(\frac{L}{L_\odot}\right)$$

↑ Absolute magnitude of our sun

$$24.93 = 2.5\log\left(\frac{L}{L_\odot}\right)$$
$$9.972 = \log\left(\frac{L}{L_\odot}\right)$$
$$\underline{L = 9.375\times10^{9}\,L_\odot}$$

$$\frac{9.375\times10^{9}\,L_\odot}{1.58\times10^{11}\,M_\odot} = \boxed{5.934\times10^{-2}\,L_\odot/M_\odot}$$

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26. (a) Wein's Law!

$$\lambda_{peak} = \frac{2900000}{T} \Rightarrow T = \frac{2900000}{\lambda_{peak}} = \frac{2900000}{562\,nm} = \boxed{5160K}$$

(b) Stefan-Boltzmann Law

Note: When everything is in convenient solar units, lots of stuff cancels and we get:

$$L = R^2 T^4 \quad \text{or} \quad R = \sqrt{\frac{L}{T^4}}$$

$$R = \sqrt{\frac{0.370 L_\odot}{((5160K)/(5778K))^4}} = \boxed{0.763 R_\odot}$$

(c) For main sequence stars:

$$L \approx M^{3.5}$$
$$0.370 = M^{3.5}$$
$$M = \boxed{0.753 M_\odot}$$

(d) $M_A - M_B = 2.5 \log\left(\frac{L_B}{L_A}\right)$

$$4.83 - M_B = 2.5 \log(0.370 L_\odot)$$
$$4.83 - M_B = -1.079$$
$$-4.83 \qquad -4.83$$
$$\boxed{5.909} = M_B$$

← Not final answer

$$m - M = -5 + 5 \log(d)$$
$$12.9 - 5.909 = -5 + 5 \log(d)$$
$$\boxed{250.1 pc} = d$$

↑ Final Answer

(e) For Main-Sequence stars,

$$\tau = 10^{10} M^{-2.5}$$

↑ mass ($M_\odot$)

$$\tau = \boxed{2.032 \times 10^{10} yr}$$

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27.(a) i. This is a fairly obscure property (I first heard about it in some paper about AGN somewhere). If the ratio of a galaxy's radio flux density to its optical flux density is greater than around 10 or so, it is radio-loud. Otherwise, it is radio quiet.

- Two of the points were for correctly stating radio-loud.
- The rest were for calculating the ratio of radio flux density to optical flux density (12.251) and stating that it was >10.

ii. $\theta_E = \sqrt{\frac{4GM}{Dc^2}}$ where everything is in SI units and θ is in radians.

$$1.86\times10^9\, M_\odot \left(\frac{1.99\times10^{30}\text{ kg}}{M_\odot}\right) = 3.70\times10^{39}\text{ kg}$$

$$1050\text{ Mpc}\left(\frac{10^6\text{ pc}}{\text{Mpc}}\right)\left(\frac{3.09\times10^{13}\text{ km}}{\text{pc}}\right)\left(\frac{10^3\text{ m}}{\text{km}}\right) = 3.245\times10^{25}\text{ m}$$

$$\theta_E = \sqrt{\frac{4(6.67\times10^{-11})(3.70\times10^{39}\text{ kg})}{(3.245\times10^{25}\text{ m})(3\times10^8\text{ m/s})^2}} = 5.814\times10^{-7}\text{ radians}$$

$$5.814\times10^{-7}\text{ rad}\left(\frac{180^\circ}{\pi}\right)\left(\frac{60'}{1^\circ}\right)\left(\frac{60''}{1'}\right) = \boxed{0.1199''}$$

In all honesty, this question was mostly testing unit skills (although the ring calculation was important too).

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(b) i. Just a friendly reminder, we're ignoring relativistic effects.

$KE = \frac{1}{2}mv^2$, $KE = \frac{1}{2}(9.109\times10^{-31}\,kg)(0.994 \times 3\times10^{8}\,m/s)^2 = 4.05\times10^{-14}\,J$

Planck's constant →
$E = \frac{hc}{\lambda}$, $\lambda = \frac{hc}{E} = \frac{(6.626\times10^{-34}\,J\cdot s)(3\times10^{8}\,m/s)}{(4.05\times10^{-14}\,J)} = 4.908\times10^{-12}\,m$

$4.908\times10^{-12}\,m\left(\frac{10^9\,nm}{m}\right) = \boxed{4.908\times10^{-3}\,nm}$

Seen in X-Ray

ii. $T = \frac{2900000}{\lambda_{peak}} = \frac{2900000}{4.908\times10^{-3}\,nm} = \boxed{5.909\times10^{8}\,K}$

iii. First, approximate the hotspot to be a sphere. And in thermal equilibrium. Use small-angle to determine radius, and use Stefan-Boltzmann to determine luminosity.

$\theta = 206265\frac{D}{d}$, $\frac{d\theta}{206265} = \frac{206265 D}{206265} = \frac{d\theta}{206265} = D$

$1050\,Mpc\left(\frac{10^6\,pc}{Mpc}\right)\left(\frac{3.09\times10^{13}\,km}{pc}\right)\left(\frac{10^3\,m}{km}\right) = 3.245\times10^{25}\,m$

$\frac{(3.245\times10^{25}\,m)(0.285'')}{206265} = 4.483\times10^{19}\,m$

$L = 4\pi r^2\sigma T^4 = 4\pi(2.242\times10^{19}\,m)^2\sigma(5.909\times10^{8}\,K)^4$
$= 4\pi(5.026\times10^{38}\,m^2)(5.67\times10^{-8}\,W/m^2K^4)(1.219\times10^{35}\,K^4)$
$= \boxed{4.365\times10^{67}\,W}$

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Remember this from (a).ii?

(c) i. $v = \sqrt{\frac{GM}{r}}$

$v^2 = \frac{GM}{r}$

$r = \frac{GM}{v^2}$ → $r = \frac{(6.67\times10^{-11})(3.70\times10^{39}\,kg)}{(3\times10^{8}\,m/s)^2} = \boxed{2.742\times10^{12}\,m}$

ii. Does the point fall within the Schwartzchild radius?

$r_S = \frac{2GM}{c^2} = 2\left(\frac{GM}{c^2}\right) = \boxed{2r = 5.484\times10^{12}\,m}$

↑ Twice the radius of the orbiting particle.

2 points for stating "within". Remaining points for justifying answer using Schwartzchild radius Equation could be solved either symbolically to get 2r or numerically to get 5.484×10^{12} m for full credit.

1 SQUARE =