68.

a. Kepler's 3rd law

$$a^3 = (M+m)P^2 = (0.95\ M_{sun})\left(\frac{120}{365.25}\ yr\right)^2 \quad \rightarrow \quad a = 0.468\ AU = 0.47\ AU$$

b. At the radius of the planet's orbit, the energy from the star is spread out on the surface of a sphere with the same radius.

$$F = \frac{energy}{area} = \frac{L_{star}}{4\pi R^2} = \frac{(0.88)\ 3.84 * 10^{26}\ W}{4\pi[(0.468\ AU)(1.496 * 10^{11}\ \ m/AU)]^2} = 5486\frac{W}{m^2} = 5500\frac{W}{m^2}$$

c. Equilibrium temperature can be calculated as found here.

$$T_{eq} = \left(\frac{L(1-\text{albedo})}{16\sigma\pi D^2}\right)^{\frac{1}{4}} = \left(\frac{(0.88)\ 3.84 * 10^{26}\ W\ (1-0.15)}{16\sigma\pi[(0.468\ AU)(1.496 * 10^{11}\ \ m/AU)]^2}\right)^{\frac{1}{4}} = 378.7K = 380\ K$$

380 K is equal to about 107° C, which is not very habitable for human life.

69.

a. Both the star and planet orbit (circularly) around the center of mass, $m_s r_s = m_p r_p$. The expression for r_s can either be derived as shown below, or the final equation can simply be used verbatim.

$$r_s = \frac{m_p r_p}{m_s} = \frac{m_p}{m_s}(r_{tot} - r_s) \quad \rightarrow \quad \left(1 - \frac{m_p}{m_s}\right) r_s = \frac{m_p}{m_s} r_{tot} \quad \rightarrow \quad r_s = \frac{r_{tot}}{(m_s/m_p)+1}$$

$$r_s = \frac{(0.29\ AU)(1.496 * 10^{11}\ \ m/AU)}{\left(\frac{0.42 * 1.99 * 10^{30}\ kg}{1.12 * 10^{25}\ kg} + 1\right)} = 5.81 * 10^5\ m$$

$$v_s = \frac{2\pi r_s}{P} = \frac{2\pi(5.81 * 10^5\ m)}{88\ days * (24 * 3600)\ s/day} = 0.48\ m/s$$

b. Doppler shift

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \quad \rightarrow \quad v = \frac{\Delta\lambda}{\lambda} c = \left(\frac{1}{1 * 10^9}\right)\left(3.00 * 10^8\ \frac{m}{s}\right) = 0.30\ \frac{m}{s}$$

The radial velocity from part (a) is greater than 0.30 m/s, so it should be detectable.

c. If a binary is not perfectly edge-on, the observed radial velocity is given by $v \sin(i)$, where i is the inclination angle.

$$v_{observed} = v \sin(30°) = \left(0.48\frac{m}{s}\right) * \frac{1}{2} = 0.24\frac{m}{s}$$

70.

a. Jeans mass can be calculated as found here.

$$3\left(\frac{M}{m}\right)kT < \frac{3}{5}\frac{GM^2}{R_c} \quad \rightarrow \quad M > \frac{5R_c kT}{Gm}$$

$$M_{Jeans} = \frac{5R_c kT}{Gm} = \frac{5(2.20 * 10^{14}\ m)(1.38 * 10^{-23}\ J/K)(15\ K)}{(6.67 * 10^{-11}\ Nm^2/kg^2)(1.673 * 10^{-27}\ kg)} = 2.04 * 10^{30}\ kg$$

This is just above 1 Msun ($1.99 * 10^{30}\ kg$), so a 5 Msun cloud will indeed collapse.

b. Conservation of angular momentum

$$I_0\omega_0 = I_f\omega_f \quad \rightarrow \quad \left(\frac{2}{5}mr_0^2\right)\left(\frac{2\pi}{T_0}\right) = \left(\frac{2}{5}mr_f^2\right)\left(\frac{2\pi}{T_f}\right)$$

Note that the constant factors of $\frac{2}{5}m$ in the moment of inertia and 2π in the angular velocity will drop out, leaving just $\frac{r_0^2}{T_0} = \frac{r_f^2}{T_f}$

$$T_f = \frac{T_0}{r_0^2}r_f^2 = \frac{1.0 * 10^6\ yr * (365.25 * 24)\ hr/yr}{(2.20 * 10^{14}\ m)^2}(1.5 * 6.96 * 10^8\ m)^2 = 0.197\ hr = 0.20\ hr$$

71.

a. Distance modulus

$$d = 10^{(m-M+5)/5} = 10^{(7.3+2.4+5)/5} = 870\ pc$$

b. The nebula is dimming the apparent magnitude by $0.2\ kpc * 1.5\frac{mag}{kpc} = 0.3\ mag$, so the actual apparent magnitude without extinction is +7.0.

$$d = 10^{(7.0+2.4+5)/5} = 760\ pc$$

72.

a. First find the total energy emitted in a year.

$$0.34\left(3.84 * 10^{26}\frac{J}{s}\right) * (365.25 * 24 * 3600)\frac{s}{yr} = 4.12 * 10^{33}\frac{J}{yr}$$

Then convert to mass via Einstein's equation, $E = mc^2$.

$$m = \frac{E}{c^2} = \frac{4.131 * 10^{33}\ J}{(3.00 * 10^8 m/s)^2} = 4.58 * 10^{16}\ kg$$

b. Main sequence lifetimes for stars less massive than 1 Msun are given by $\frac{\tau}{\tau_{sun}} = \left(\frac{M}{M_{sun}}\right)^{-2.5}$. The MS lifetime of the Sun is about 10 Gyr.

$$\tau = (0.82)^{-2.5} * \tau_{sun} = 1.64 * (10\ Gyr) = 16.4\ Gyr$$